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A COMPARATIVE STUDY BETWEEN THE SUCCESSIVE SCREW DISPLACEMENT AND QUATERNION BASED METHODS USED IN FORWARD KINEMATICS OF SERIAL ROBOT MANIPULATOR

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ABSTRACT

In this article, we compare two forward kinematic formulation procedures for the 6DoF serial robot manipulator. Both formulation procedures are based on the screw theory. Screw theory based methods are alternative to the classical methods such as D-H (Denavit-Hartenberg) based conventions. The following two methods, successive displacement method and quaternion based method have been considered for comparison in this article. Compared to the successive screw displacement method, quaternion method is computationally efficient and compact. Kinematic modelling of both methods are presented with case studies and compared, concerning forward kinematics of the serial robot manipulator.

KEYWORDS: Forward Kinematic, Successive Displacement & Robot Manipulator

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Original Article

INTRODUCTION

Kinematics is a fundamental part for the motion devices such as robots. The kinematic analysis of the robot manipulator provides the relationship between the end-effector positions and joint displacements. The forward kinematic analysis is the method of determining the position of end-effector, from the position of joints given. Since, the solutions of kinematic analysis are easy to attain and require very less number of computations, compared with equations of dynamics.

Various methods have been used in robot kinematic study and screw theory is one of the significant method among them. Screw theory based solutions have been using in many applications of robotics for last few years. Robert S. Ball has developed the complete screw theory which is traced form the theorems of Chasales, developed by Chasles Poinsot in 1800[1]. In the screw theory every rigid body transformation, with respect to a coordinate system of reference frame can be described by displacement of screw, which is translation along the axis of rotation and by angle ϕ about the same axis[2]. There are two important advantages of using screw theory for describing the kinematics of the rigid body. The first one is that it permits rigid body global description motion that helps to avoid singularities due to the use of local coordinates system. The second one is that it provides a geometric description of the rigid body motion which simplifies the mechanisms[3].

In this paper, we presented a comparative study for the forward kinematics formulation methods in which both are based on the screw theory. In these methods first one is based on the successive screw displacement method and the second one is based on the quaternion algebra. These two methods are given in [4] and[5]are extensively developed in formulations of mathematics and analysed in details. Additionally, case studies have been established for the both

methods using 6DoF robot manipulator.

SCREW THEORY

Screw theory is a geometric entity which characterises both translational and rotational quantities. It is poised of an axis, on which both translational and rotational quantities are defined, a scalar pitch which correlates both quantities[6].

Instantaneous motion of rigid body is represented by a twist which is relative to a reference frame. It is typically denoted with Plucker coordinates as $\xi = (\omega \ ; v \ )^T$, where ω is the angular velocity of the rigid body about a certain axis and ***v*** is the instantaneous linear velocity of a point **"O",** coincident with the origin. *v* has a component parallel to the axis another component is normal to the twist axis. This twist can be transformed into its magnitude $\dot{p}$ and normalised screw $\hat{\xi}$. **Figure.1a** and **Figure.1b** represents the components of a screw and screw displacement respectively, where k denotes the screw axis and k_o is a position vector of a point in axis of screw. According to Figure.1a, ξ representation shown in **Equation.(1)**

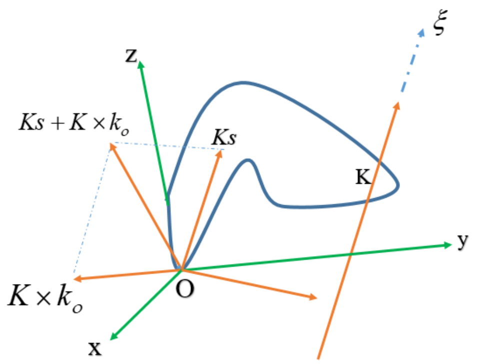

Figure.1a: Components of Screw

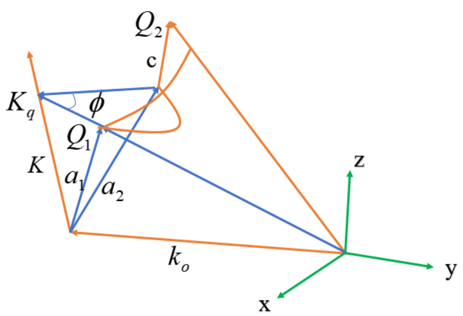

Figure.1b: Displacements of Screw

Translation and rotation are related by the screw pitch **s:**

$$\xi = \begin{bmatrix} K \\ Ks + K \times k_o \end{bmatrix} = \begin{bmatrix} k \\ ks + k_o \times k \end{bmatrix} \dot{p} = \hat{\xi}\dot{p} \tag{1}$$

Particularly there are two cases. When the motion is translational $s = \infty$ and so $\hat{\xi} = (0; k)^T$. For rotational

motion it is assumed that $s = 0$,and the normalized screw reduces to $\hat{\xi} = (0; k \times k)^T$.

Along the screw axis helical displacement can represented by parameters of Rodrigues, as shown in **Figure.1(b).** These correspond to the k and k_o, which defines axis of screw, the linear displacement *c* parallel to it and angular displacement ϕ along the axis. The transformation matrix representation using these parameters expressed as shown in **Equation. (2).** This displays a screw that can be employed to define the rigid body position relative to a reference frame.

$$M = \begin{bmatrix} E(\phi) & q(c) \\ 0 & 1 \end{bmatrix} \tag{2}$$

$$E(\theta) = \begin{bmatrix} t_\phi + k_x^2(1-t_\phi) & s_y s_x(1-t_\phi) - k_z k_\phi & k_z k_x(1-t_\phi) - t_y t_\phi \\ s_y s_x(1-t_\phi) - k_z k_\phi & t_\phi + k_x^2(1-t_\theta) & s_y s_z(1-t_\phi) - k_x k_\phi \\ k_z k_x(1-t_\phi) - t_y t_\phi & s_y s_z(1-t_\phi) - k_x k_\phi & t_\phi + k_z^2(1-t_\phi) \end{bmatrix} \tag{3}$$

$$q(c) = ck + [I - E(\phi)]k_0 \tag{4}$$

A rigid body may undergo several displacements of screw and it can be denoted by an equivalent displacement of screw. Multiplication of two homogeneous matrices of each displacement results in the equivalent homogeneous matrix of screw displacement [7].

Forward Kinematic Modelling By the Using Successive Screw Displacement Method

Considering a kinematic chain in which links are connected by various joints. So, the link *i* movement, relative to the predecessor link *i-1,* which is connected by a joint is denoted by a screw ξ_i. The positon of a link relative to its predecessor is represented by the transformation matrix$^{i-1}M_i$, founded as in **Equation.(2),** from the parameters of the joint displacement.

The position of a link *h* to a link *g* is represented by successive displacements of screw made by each joint in the sub-chain between *h* and *g*. The overall displacement is attained by homogeneous transformation matrix$^{i-1}M_i$ pre multiplication as represented in **Equation.(5).** It is considered that, link g is successively displaced according to the movements of screw, from the joint nearest to the link *h* to the joint nearest to link *g:*

$${}^{g}M_h = {}^{g}M_1 \, {}^{1}M_2 \ldots\ldots\ldots {}^{n-1}M_n \, {}^{n}M_h \tag{5}$$

Representations of screw are always relative to the system of reference frame coordinate. We can determine the kinematic equation of the robot manipulator, using any reference system. Once determined the screw in a system of referential frame, a transformation ${}^{i}T_j$ should implement to attain its expression in different reference frame, ${}^{i}T_j$ represented in **Equation. (6),** where $\zeta({}^{i}q_j)$ is the skew-symmetric matrix from the positon vector between them ${}^{i}E_j$ and origins of two system. Where ${}^{i}E_j$ is the rotation matrix from the system *j* to system *i,* as represented in the **Figure.2:**

$$^{i}\xi = {}^{i}T_{j}\ {}^{j}\xi = \begin{bmatrix} {}^{i}E_{j} & 0 \\ \zeta({}^{i}p_{j})\,{}^{i}E_{j} & {}^{i}E_{j} \end{bmatrix} \tag{6}$$

Case Study

In this case study a 6 DoF robot manipulator, modelled using successive screw displacements procedure**.** **Figure.2a** represents the schematic diagram of the elbow robot manipulator. In this robot manipulator, the second joint axis perpendicularly intersect with the first joint axis, the second joint axis is parallel to the third and fourth joint axis, the fourth joint with a small is offset distance l_4 is perpendicular to the fifth joint axis, and the fifth joint is perpendicular to the sixth joint axis. Firstly, reference configuration with respect to which the manipulator displacement should be measured. The Reference configuration has shown in **Figure.2b**where the first joint axis**,** ξ_1, vertically points up in the direction of Z; the second(ξ_2),third(ξ_3), and fourth(ξ_4) joint axes, are all directing out of the paper. The fifth joint axis(ξ_5) directing in the positive direction z-axis. The coordinate of system of hand is located at Q point such that w_0-axis directs in the positive direction of x-axis and u_0-axis directs in the positive direction of z-axis. At this position, the locations of the screw axes with respect to the frame of fixed reference are given below;

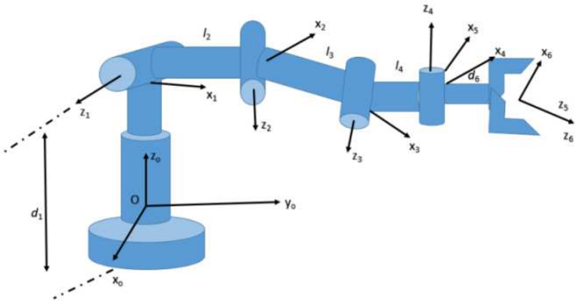

Figure.2a: 6 DOF Elbow Manipulator

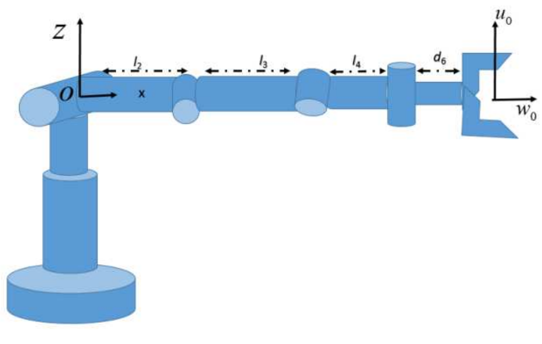

Figure.2b: Reference Position of the Elbow Manipulator

$$joint1: k_i(0,1,1); k_{oi}(0,0,0)$$
$$joint2: k_i(0,-1,0); k_{oi}(0,0,0)$$
$$joint3: k_i(0,-1,0); k_{oi}(l_2,0,0)$$
$$joint4: k_i(0,-1,1); k_{oi}(l_2+l_3,0,0)$$
$$joint5: k_i(0,0,1); k_{oi}(l_2+l_3+l_4,0,0)$$
$$joint6: k_i(1,0,0); k_{oi}(0,0,0)$$

The reference positions of the end effector are given;

$$u_0 = [0,0,1]^T$$
$$v_0 = [0,-1,0]^T$$
$$w_0 = [1,0,0]^T$$
$$q_0 = [l_2+l_3+l_3,0,0]^T$$

Now, end effector target position is given by

$$u = [u_x, u_y, u_z]^T$$
$$v = [v_x, v_y, v_z]^T$$
$$w = [w_x, w_y, w_z]^T$$
$$q_0 = [q_x, q_y, q_z]^T$$

Now, substituting the joint axes coordinates in to **Equation. (5).** We obtain the transformation matrices;

$$^0M_1 = \begin{bmatrix} t\phi_1 & -k\phi_1 & 0 & 0 \\ k\phi & t\phi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^1M_2 = \begin{bmatrix} t\phi_2 & 0 & -k\phi_2 & 0 \\ 0 & 1 & 0 & 0 \\ k\phi_2 & 0 & t\phi_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^2M_3 = \begin{bmatrix} t\phi_3 & 0 & -k\phi_3 & l_2(1-t\phi_3) \\ 0 & 1 & 0 & 0 \\ k\phi_3 & 0 & t\phi_3 & -l_2k\phi_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^3M_4 = \begin{bmatrix} t\phi_4 & 0 & -k\phi_4 & (l_2+l_3)(1-t\phi_4) \\ 0 & 1 & 0 & 0 \\ k\phi_4 & 0 & t\phi_4 & -(l_2+l_4)k\phi_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^4M_5 = \begin{bmatrix} t\phi_5 & -k\phi_5 & 0 & (l_2+l_3+l_4)(1-t\phi_5) \\ k\phi_5 & t\phi_5 & 0 & -(l_2+l_3+l_4)k\phi_5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^5M_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & t\phi_6 & -k\phi_6 & 0 \\ 0 & k\phi_6 & t\phi_6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The matrix products $^1M_2\,{}^2M_3\,{}^3M_4 and {}^0M_1\,{}^1M_2\,{}^2M_3\,{}^3M_4$ are computed as

$$
{}^{1}M_{2}{}^{2}M_{3}{}^{3}M_{4} = \begin{bmatrix} t\phi_{234} & 0 & -k\phi_{234} & l_2 t\phi_2 + l_3 t\phi_{23} - (l_2 + l_3)(t\phi_{234}) \\ 0 & 1 & 0 & 0 \\ k\phi_{234} & 0 & t\phi_{234} & l_2 k\phi_{234} + l_3 k\phi_{23} - (l_2 + l_3)(k\phi_{234}) \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
M_{1}{}^{1}M_{2}{}^{2}M_{3}{}^{3}M_{4} == \begin{bmatrix} t\phi_1 t\phi_{234} & -k\phi_1 & -t\phi_1 k\phi_{234} & t\phi_1[l_2 t\phi_2 + l_3 t\phi_{23} - (l_2 + l_3) t\phi_{234}] \\ k\phi_1 t\phi_{234} & t\phi_1 & -t\phi_1 k\phi_{234} & k\phi_1[l_2 t\phi_2 + l_3 t\phi_{23} - (l_2 + l_3) t\phi_{234}] \\ k\phi_{234} & 0 & t\phi_{234} & [l_2 k\phi_2 + l_3 k\phi_{23} - (l_2 + l_3) k\phi_{234}] \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

Quaternion Based Forward Kinematic Modelling for the 6DOF Robotic Manipulator

Quaternion

Quaternions are rank 4 hyper complex numbers, representing a four dimensional vector space over real numbers field [20,21]. The quaternion can be denoted in the form;

$$
p = (p_o, p_V) \tag{7}
$$

Where p_o is scalar quantity and p_V is a vector. If $p_o = 0$ then, we have pure quaternion. The two quaternions sum and product given as,

$$
p_a + p_b = (p_{a0} + p_{bo}), (p_{aV} + q_{bV}) \tag{8}
$$

$$
p_a \otimes p_b = (p_{a0} p_{b0} - p_{aV} \cdot p_{bV}), (p_{a0} p_{bv} + p_{b0} p_{aV} + p_{aV} \times p_{bv}) \tag{9}
$$

Where $\otimes$ represents the quaternion product. Norm, inverse and conjugate of the quaternion can be represented in the forms

$$
\|p\|^2 = p \otimes p^* = p_0^2 + p_1^2 + p_2^2 + p_3^2 \tag{10}
$$

$$
p^{-1} = \frac{1}{\|p\|^2} p^* and \|p\| \neq 0 \tag{11}
$$

$$
p^* = (p_1, -p_V) = (p_0, -p_1, -p_2, -p_3) \tag{12}
$$

that satisfies the $p^{-1} \otimes p = p \otimes p^{-1} = 1$ when $\|p\|^2 = 1$,we get quaternion unit that satisfies the $p^{-1} = p^*$.

Screw Theory Using Quaternion

If the motion of screw axis does not pass through origin as represented in the **Figure.3a**then, the motion of the screw is given by

$$T=\begin{bmatrix} I_{3\times3} & q \\ 0 & 1 \end{bmatrix}\begin{bmatrix} E(\phi,d) & \frac{\phi}{2\pi}ld \\ 0 & 1 \end{bmatrix}\begin{bmatrix} I_{3\times3} & -q \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} E(\phi,d) & \frac{\phi}{2\pi}ld+(I_{3\times3}-E(\phi,d))q \\ 0 & 1 \end{bmatrix} \tag{13}$$

In Eq. (13), screw motion is given by using homogenous transformation matrices (4×4). It employs sixteen parameters while it needed only six parameters. By using quaternion algebra, we can represent screw motion in a more compact(efficient) form than homogenous transformation matrices. If we separate motion of screw as a translation and rotational then, we have

Translation

$$\frac{\phi}{2\pi}ld+(I_{3\times3}-E(\phi,d))q \tag{14}$$

and

Rotation: $E(\phi,d)$

Now by using quaternions, we can express these equations as follows;

Translation

$$c=\tilde{p}+q-p\otimes q\otimes p^{*} \tag{15}$$

Where p is the position vector and $\tilde{p}$ is amount of pure translation.

Formalisation of Kinematic Modelling Using Quaternion Based Method

In this method first we have to determine the joint axis vectors; First we attach axis vector which describes the joint motion. Later we have to obtain the transformation operators: these operators for all joints can be attained by using quaternions, as follows

Translation

$$p_i^{0}=q_i-p_i\otimes p_i\otimes q_i^{*} \tag{16}$$

Rotation: $p_i=\sin(\frac{\phi_i}{2})d_i,\cos(\frac{\phi_i}{2})$

Where q_i represents the arbitrary point on the i^{th} axis.

Now,

From **Equation. (15) and (9)** Rigid transformation of robot manipulator can be given as

Position

$$q_{hq+1}=p_{1n}\otimes q_{hq}\otimes p_{1n}^{*}+p_{1n}^{o} \tag{17}$$

Orientation: $r_{0+1} = p_{1n} \otimes r_0 \otimes p_{1n}^*$

Where r_0, q_{hq} denotes the end effector orientation and position before the transformation and r_{0+1} , q_{hq+1} denotes the end effector orientation and positon after the transformation.

Case Study

In this case study forward kinematic problem solved for the elbowserial robot manipulator using quaternion methods.

Firstly, determination of all joints axes

$$\begin{aligned} d_1 &= d_4 = [0,0,1] \\ d_2 &= d_5 = [0,1,0] \\ d_3 &= d_6 = [1,0,0] \end{aligned} \tag{18}$$

Any point on these axes can be given as

$$\begin{aligned} q_1 &= q_2 = [0,0,r_0] \\ q_3 &= [r_0,0,r_0] \\ q_4 &= q_5 = q_6 = [r_1 + r_2,0,1] \end{aligned} \tag{19}$$

Secondly, the operators of transformation which are in quaternion can be given by using **Equation.(16).** At end the forward kinematic equation can be attained as follows,

Position

$$p_{16} \otimes q_6 \otimes p_{16}^* + p_{16}^o \tag{20}$$

Orientation:

$$p_{16} \otimes r_6 \otimes p_{16}^*$$

$$\begin{aligned} p_1^o &= q_1 - p_1 \otimes q_1 \otimes p_1^* \\ p_{1i}^o &= p_{1i-1} \otimes q_{i-1} \otimes p_{i-1}^* - p_{1i} \otimes q_i \otimes p_{1i}^* + p_{i-1}^o \\ p_{16} &= p_1 \otimes p_2 \otimes p_3 \otimes p_4 \otimes p_5 \otimes p_6 \otimes \end{aligned} \tag{21}$$

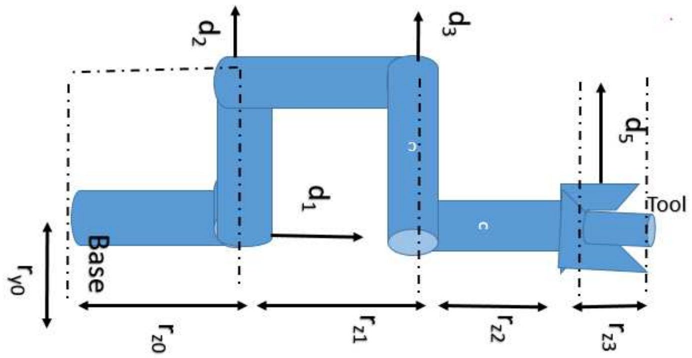

Figure.3a: Reference Configuration of 6Dof Serial Arm Robot Manipulator

Comparing the Methods

Both successive screw displacement method and quaternion methods are systematic for identification of parameters and modelling of positional kinematics. The reference choice flexibility in successive screw displacement method is a significant feature, since it facilities the identification of parameter process and can be employed to attain simplified formulations. Parameter identification requires only two frames for the entire system. Only, one end effector coordinates system and one fixed coordinate system is required, we can choose the reference positon arbitrarily. In the case of forward kinematic analysis Joint variables represent actual displacements and derived by composition of homogeneous transformation matrices.

Quaternions are compact and convenient mathematical representations, for expressing the rigid body's attitude in three dimensions. Some of the advantages provided by quaternion based method are concatenating rotations are numerically more stable, singularity free, computationally faster, and unambiguousness. Extracting axis of rotation and angle is easy, Interpolation is straighter forward. Compared with the successive screw displacement method, quaternion based method solutions are computationally more efficient and they requires less storage area. This work is focused on serial robot manipulators, it is should be consider that use of screw based methods are straight forward for multi-robot systems and parallel manipulators. In detail, Davies method is appropriate application only to closed kinematic chains, which are inherent for multi-robot systems and can be identified in parallel manipulators.

CONCLUSIONS

This work is investigated two types of forward kinematic modelling of 6DoF robot manipulators using case studies. The both screw based methods are less known but has few advantages in designing and modelling of kinematic chains. Both successive screw displacement method and quaternion based methods were applied to the same kind of robot manipulator model to simplify their comparison. It was observed that quaternion based method, apparently more intricate, is more flexible. The comparison made pointed that quaternion based method has advantages over the successive displacement modelling method in some perspectives.

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